

A Simple Model of Bipedal Walking Predicts the Preferred Speed–Step Length Relationship

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We used a simple model of passive dynamic walking, with the addition of active powering on level ground, to study the preferred relationship between speed and step length in humans. We tested several hypothetical metabolic costs, with one component proportional to the mechanical work associated with pushing off with the stance leg at toe-off, and another component associated with several possible costs of forcing oscillations of the swing leg. For this second component, a cost based on the amount of force needed to oscillate the leg divided by the time duration of that force predicts the preferred speed–step length relationship much better than other costs, such as the amount of mechanical work done in swinging the leg. The cost of force/time models the need to recruit fast muscle fibers for large forces at short durations. The actual mechanical work performed by muscles on the swing leg appears to be of relatively less importance, although it appears to be minimized by the use of short bursts of muscle activity in near-isometric conditions. The combined minimization of toe-off mechanical work and force divided by time predicts the preferred speed–step length relationship. [DOI: 10.1115/1.1372322]

1 Introduction

Humans tend to choose a step length or step frequency that minimizes metabolic energy consumption at a given walking speed [1–3]. The step length, s , has been found empirically to obey the power law $s \sim v^\beta$, where v is the walking speed. The value of β is around 0.42 for adults [4]. Metabolic cost depends on both step length and step frequency [5], and deviating from this power law while maintaining the same speed will result in an increase in the net metabolic energy consumption E , normalized by body weight and distance traveled (see Fig. 1).

A number of relationships have been proposed to explain these power laws. Cavagna and Margaria [6] showed that during the single leg stance phase, an inverted pendulum model for the stance leg can explain exchange between kinetic and gravitational potential energy of the center of mass, with additional energy provided by the muscles performing work on the center of mass, termed *external work*. A significant amount of work, termed *internal work*, must also be performed to move the limbs relative to the center of mass; both external and internal work, measured empirically, are positively correlated with walking speed [7]. As a determinant of optimum step frequency, however, the minimization of total mechanical work makes predictions that are about 20–30 percent too low [8] or requires separate models for low or high speeds [9,10]. Several investigators have pointed out drawbacks to the empirical estimation of internal work that could contribute to these discrepancies. For example, internal work calculations require a number of assumptions regarding the transfer of energy between segments [11–12,9]. Nevertheless, it is clear that external work is a major determinant of step frequency at slow speeds, while at fast speeds the movement of the limbs is more significant [9].

Another approach is to calculate mechanical work theoretically. Minetti and Alexander [13] proposed a sophisticated model of limb dynamics that could predict work of walking and running (see also [14]). Based on an empirical metabolic rate function, their model could predict the approximate speed–step length relationship. Despite some drawbacks such as an unrealistically

long leg length (1.7 times anatomical) and an overly low prediction of metabolic cost, this model is promising because it demonstrates a need to consider limb dynamics.

Simpler models, restricted to walking and based purely on principles of mechanics, can provide insight to the determinants of metabolic energy consumption. Alexander used a simple point mass model to show that positive work in walking is necessary to restore energy lost during heel strike [15,16]. Mochon and McMahon [17] showed that the ballistic motion of the swing leg closely resembles that observed in humans, and McGeer [18] showed that, with appropriate dynamic parameters, the entire step cycle, including the impact of the foot with the ground, can be explained by passive dynamics with energy provided by a slight downhill slope or by active power provided by a hip torque or an impulse at toe-off [19]. Garcia et al. [20,21] simplified the gravity-powered model to an irreducible limit, and showed that as the inertia of the legs approaches zero relative to that of a point mass pelvis and torso, a passively stable gait is retained even though the only parameter is the slope. Their “simplest walking model” has the same energetic features as Alexander’s [15] heel strike model, but can also produce an entire periodic gait cycle.

A modification to the simplest model of walking mechanics yields several insights regarding energetics [22]. This model retains as the analytical tractability of Garcia et al.’s [20] model, but adds the ability to walk on level ground in the manner of McGeer [18]. The model shows that mechanical energy is lost with the impact of the swing leg with the ground at heel strike, and that the most efficient way to restore this energy is to push off with the trailing stance leg immediately before heel strike. Although it is inefficient to add net energy to a step through torques applied at the hip, there is some advantage to using hip muscles to produce forced oscillations of the swing leg. Forced oscillations, applied with an effective torsional spring, can increase the step frequency and decrease the mechanical energy lost at heel strike.

We presently apply the same model [22] to evaluate simple hypotheses for metabolic cost of muscle activity. These include the hypothesis that metabolic cost is proportional to the amount of mechanical work performed, the peak force generated by muscle, the peak force divided by burst duration, and several others. The simplest model, and slightly more complex variations more similar to that of McGeer [18], are useful for exploring the fundamental mechanics of walking as well as the qualitative consequences

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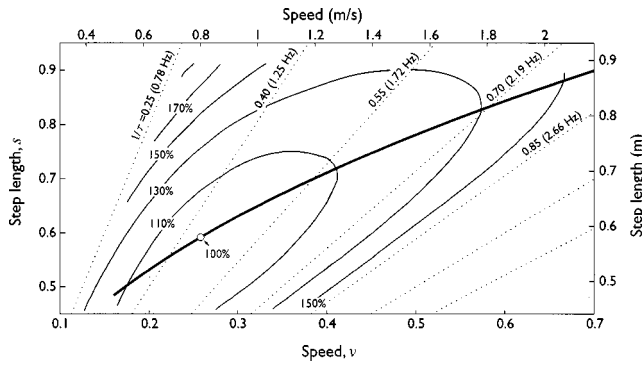


Fig. 1 Metabolic energy costs as a function of speed, v , and step length, s . Data are from [1], replotted in the manner of [2]. Also shown is the preferred speed-stride length relationship $s \sim v^{0.42}$ (thick solid line) reported by [4]. Energy contours are shown as a percentage of a nominal gait, with the cost of standing subtracted. Note that there is a sharp increase in energy consumption if speed is increased while keeping step length constant. At fast speeds and short step lengths, energy consumption appears to be dominated by step frequency, plotted as dotted line contours (τ is the dimensionless step period). Top and right axes are in actual SI units. Bottom and left axes are in dimensionless units: Step length is normalized by leg length $l=0.98$ m, and speed is normalized by \sqrt{g} and is equal to the square root of a Froude number [16]. Dimensionless step period τ is normalized by $\sqrt{l/g}$.

of a hypothetical metabolic cost model. The extreme simplicity of the model should also clearly reveal underlying principles of the energetic cost of transport, a task difficult to achieve when using a large number of empirical parameters.

2 Simple Models of Walking

We use three variations of walking models to study hypothetical metabolic costs. All are based on the principles of passive dynamic walking, but with the addition of actuation for walking on level ground. These quasi-passive models include two variations of [22], termed the Idealized Simple Model (ISM, see Fig. 2(a)) and the Simple Model (SM, see Fig. 2(b)), as well as a model more similar to that of McGeer [18] with more realistic inertial characteristics, termed the Anthropomorphic Model (AM, see Fig. 2(c)). The Idealized Simple Model provides analytical predictions but relies on linearization and other approximations. The Simple Model is identical except that it retains all nonlinearities. It requires numerical computations but makes it possible to test the consequences of the ISM approximations. Finally, the Anthropomorphic Model is used to test whether the fundamental principles of the ISM apply to a nonlinear model that is more physically realistic. We will use dimensionless variables throughout, with the following base units; overall mass M , leg length l , and the gravitational constant g . Time is therefore normalized by $\sqrt{l/g}$.

All three models include rigid stance and swing legs connected by a hinge joint at the pelvis and constrained to planar motion (see Fig. 2). Actuation is provided in the form of an impulse P directed along the stance leg and a springlike hip torque acting between the legs. The toe-off impulse P is applied instantaneously before heel strike, which is modeled as an instantaneous and perfectly inelastic collision that sets the initial conditions for the following step. The hip torque can either be produced by a torsional spring of stiffness k or by impulsive torques that occur at the beginning and end of the swing phase, as long as both yield the equivalent swing leg natural frequency $\omega \equiv \sqrt{k+T}$.

The ISM and SM are irreducibly simple, with legs of zero mass connecting a massive pelvis of mass M and point mass feet of mass m , taking the limit as m/M approaches zero [22].

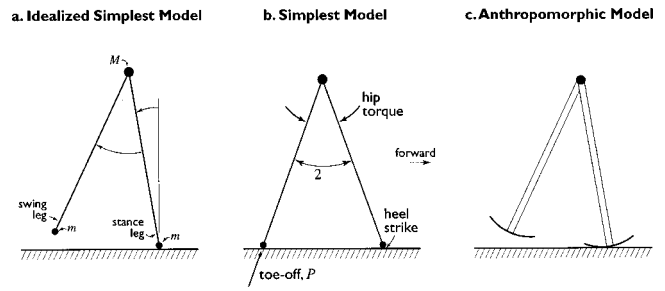


Fig. 2 Three models of walking: (a) The Idealized Simplest Model (ISM) consists of point mass pelvis and feet connected by massless legs, after Garcia et al. [20]. It employs a linearized analytic solution to the equations of motion. (b) The Simplest Model (SM) is identical to the ISM except that the equations of motion are solved numerically. (c) The Anthropomorphic Model (AM) is similar to that of McGeer [18], with legs with more realistic inertial parameters as well as curved feet to improve efficiency. All three models are powered on level ground by an impulsive push P along the stance leg applied at toe-off, as well as a springlike hip torque between the legs [22]. The SM is used to test the linearizing assumptions of the ISM, while the AM tests the idealized inertial parameters.

Analysis of the ISM yields three simple multiplicative power laws relating actuation variables P and ω to the resulting speed v , step period τ , and step length described by the initial leg angle α (see Fig. 2). From Kuo [22],

$$v \sim \omega^{1/2} P^{1/2} \quad (1a)$$

$$\tau \sim \omega^{-1} \quad (1b)$$

$$\alpha \sim \omega^{-1/2} P^{1/2}. \quad (1c)$$

These approximations were found to apply to both the SM and AM.

3 A General Model of Metabolic Costs

(a) Two Components of Metabolic Cost. We propose a general model of metabolic cost of transport (cost normalized by body weight and distance traveled), E , that has a component corresponding to the metabolic cost of pushing off with the stance leg (i.e., toe-off), E_{toe} , and a component corresponding to forced motion of the swing leg, E_{swing} :

$$E \sim E_{toe} + E_{swing}. \quad (2)$$

This general model can be used to examine the implications of several physiological hypotheses for the cost of tuning the swing leg. A significant feature of this model is that when used with the idealized simple model (ISM) to predict the speed exponent β , the prediction is not dependent on the relative proportionality between E_{toe} and E_{swing} in the overall metabolic cost, nor on any other free parameters. When applied to the (nonideal) simplest model (SM) and anthropomorphic model (AM), the sensitivity to this same relative proportionality remains very low, so that the predictions are quite robust to parameter variations.

The first component of the overall metabolic cost is hypothesized to be directly proportional to the amount of toe-off mechanical work performed on the center of mass, W_{toe} , which is itself equal to the amount of negative work performed during heel strike (see Fig. 3(a)). The ISM yields a very simple approximation [22],

$$E_{toe} \sim W_{toe} \sim \alpha v^2 \sim \omega^{-1} v^3, \quad (3)$$

where E_{toe} is the cost of transport.

For the second component of metabolic cost it is clear (see Fig. 3(a)) that there must be a cost associated with high stiffness k —otherwise the optimum step length would be very low. Be-

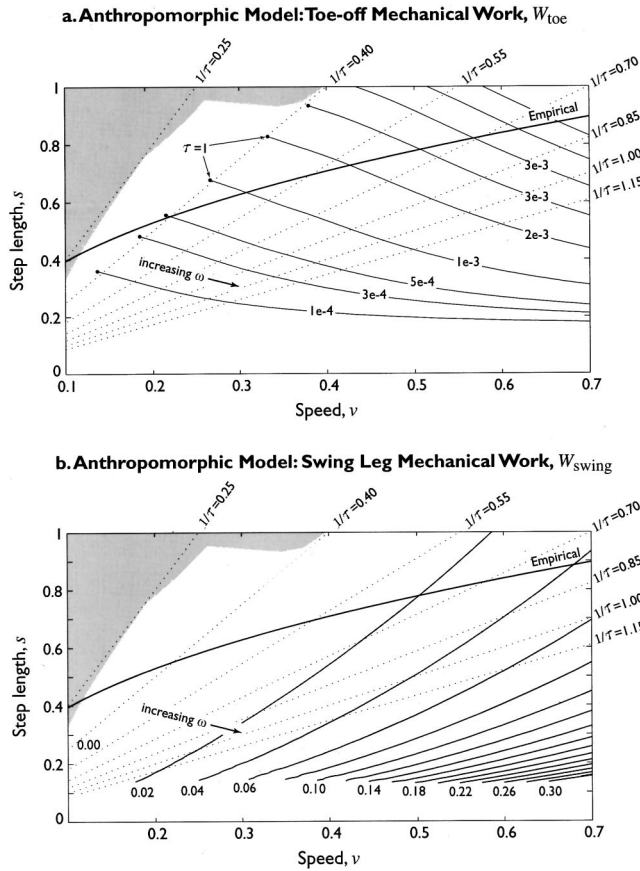


Fig. 3 Mechanical work performed: (a) by toe-off impulses on the stance leg, and (b) by hip torques on the swing leg, as a function of speed v and step length, s , using the Anthropomorphic Model. (a) Lines of constant (dimensionless) toe-off mechanical work per distance, W_{toe} (solid lines, with energy levels as labeled), increase with both speed and step length. For a given toe-off impulse, the effect of increasing the natural frequency τ of the swing leg is to increase speed at slightly shorter step lengths. Walking becomes less costly because the hip spring decreases collision losses. Without a cost assigned to tuning the swing leg, there is no obvious relationship between mechanical work and the preferred speed-step length relationship (thick solid line from Fig. 1). Shaded regions denote unstable gaits. (b) Swing leg mechanical work per distance, W_{swing} , increases sharply with step frequency ($1/\tau$, denoted by dotted lines). No linear combination of (a) and (b) predicts the preferred speed-step length relationship, indicating that other factors may contribute to cost of transport.

cause there are many possible ways to model this cost, we use a general power law in terms of two actuation parameters ω and P . We will show that a cost of the form

$$E_{swing} \sim \omega^\kappa P^\lambda, \quad (4)$$

where E_{swing} is also normalized by body weight and distance traveled, can be used to approximate a number of hypothesized costs.

Such a model can also readily predict the speed exponent β . Putting E_{swing} in terms of ω and v by combining Eqs. (1)–(4),

$$E = E_{toe} + E_{swing} \sim \omega^{-1} v^3 + c \omega^{\kappa-\lambda} v^{2\lambda}, \quad (5)$$

where c is the relative proportionality between the two terms. The step frequency that minimizes E for any speed is found by taking the partial derivative

$$\frac{\partial E}{\partial \omega} \sim -\omega^{-2} v^3 + c(\kappa-\lambda) \omega^{\kappa-\lambda-1} v^{2\lambda} = 0, \quad (6)$$

because ω is proportional to step frequency [22]. Solving Eq. (6) for ω yields

$$\omega \sim v^{(3-2\lambda)/(\kappa-\lambda+1)}, \quad s \sim v^{(\kappa+\lambda-2)/(\kappa-\lambda+1)}, \quad \beta = \frac{\kappa+\lambda-2}{\kappa-\lambda+1}, \quad (7)$$

where β is the predicted speed exponent ($s \sim v^\beta$). Note that c drops out from the prediction, so that a hypothesized cost of tuning the swing leg, in terms of κ and λ , leads directly to the predicted β with no dependence on other parameters.

(b) Hypothesized Metabolic Costs for Tuning the Swing Leg.

We will consider four physiological hypotheses for E_{swing} . These include a cost directly proportional to the work performed in actuating the swing leg, one proportional to the peak force produced by the hip muscles, one proportional to the peak force multiplied by the duration of that force, and one proportional to the peak force divided by the duration that the force is applied. (When referring to swing leg actuation, we will use force and torque interchangeably, assuming that the hip moment arm is constant.)

The amount of work performed by the hip muscles depends on tendon compliance and the shortening velocities over which they are activated. An upper bound on the work performed by the hip spring in our model is its peak potential energy. Termed *swing work*, it is proportional to internal work in the simple models (ISM and SM), but in the Anthropomorphic Model (AM) it also includes work done by the hip spring on the center of mass (see Fig. 3(b)). This proportionality holds true whether we consider only the positive work done, or any weighted sum of positive and (absolute value) of negative work, as long as the proportionality between the two is held constant. The associated metabolic cost of transport is proportional to swing work,

$$\text{Swing work: } E_{swing} \sim \frac{1}{2} \cdot \frac{k\alpha^2}{2\alpha} \sim k\alpha. \quad (8)$$

Another possibility is that E_{swing} is associated more with the amount of force generated than with the amount of work performed. The hip spring generates maximal force at a point where it performs zero work, and for muscles there is obviously a metabolic cost for exerting a force even when no work is performed [19]. A corresponding normalized cost per distance is

$$\text{Peak force: } E_{swing} \sim \frac{k\alpha}{2\alpha} \sim k. \quad (9)$$

Another possibility would be to integrate the force over time. This is equivalent to the impulse, which in turn is proportional to the momentum change induced by muscle. The normalized cost per distance is

$$\text{Impulse: } E_{swing} \sim \frac{k\alpha\tau}{2\alpha} \sim k\tau. \quad (10)$$

If the time during which the hip muscles are active is sufficiently short, it is possible that faster and therefore less metabolically efficient muscle fibers must be recruited. Kram and Taylor [23] hypothesized that the metabolic cost (in their case, for weight support) is proportional to the peak muscle force multiplied by an economy that is inversely proportionally to burst duration, and showed that such a law predicts well the metabolic cost of running at different speeds. In the case of walking, we apply a similar cost to a different situation, in which actuation of the swing leg models the short burst of nearly isometric hip muscle activity during the swing phase. Assuming the burst duty factor to be constant,

$$\text{Force/time: } E_{swing} \sim \frac{k\alpha}{2\alpha} \cdot \frac{1}{\tau} \sim k/\tau. \quad (11)$$

Such a cost increases sharply with short burst durations [22].

Table 1 Predicted speed exponents β and sensitivities (sens.) for various hypothesized metabolic costs of tuning the swing leg, E_{swing} . Sensitivity refers to the percent change in β associated with a percent change in c (a free parameter equal to the proportionality between E_{toe} and E_{swing}).

Cost of swing leg	Idealized Simple Model (ISM)		Simplest Model (SM)			Anthropomorphic Model (AM)		
	E_{swing}	β	E_{swing}	β	sens.	E_{swing}	β	sens.
Swing work	$\omega^{1.5} P^{0.5}$	0	$k\alpha$	0.25	-7.9e-3	$k\alpha$	0.090	3.5e0
Impulse	$\omega^1 P^0$	-0.50	$k\tau$	-0.088	-6.3e-1	$k\tau$	-0.28	3.5e-2
Peak force	$\omega^2 P^0$	0	k	0.26	3.0e-1	k	0.21	9.8e-1
Force/time	$\omega^3 P^0$	0.25	k/τ	0.41	2.7e-3	k/τ	0.43	1.5e-1

4 Predicted Speed Exponents

All of these cost hypotheses can be approximated by Eq. (3) and then applied to the three successively more complex models. We first use the idealized simple model (ISM) to make a rough analytic prediction of the speed exponent. The necessary approximation is made by writing k in terms of ω and taking only the leading term:

$$k = \omega^2 - 1 \approx \omega^2 \quad (12)$$

which is reasonable for large values of ω . The values of κ and λ , as well as the predicted exponent β for each of these hypotheses, are listed in Table 1. It is evident that the swing work and peak force hypotheses, with a predicted exponent of $\beta=0$, should heavily favor increasing step frequency over increasing step length as speed increases, and that the force/time hypothesis gives a prediction closer to the empirical value.

We tested these metabolic costs on the Simplest Model (SM) and the more realistic Anthropomorphic Model (AM). The simulations for a wide range of step lengths and speeds (see Table 1 and Fig. 4) were used to calculate minimum energy step length versus speed curves. For both models, there is some dependence of the predicted speed exponent β on the proportionality c of (24). The value of c was chosen so that, where possible, the predicted speed-step length curve for each cost function intersected the

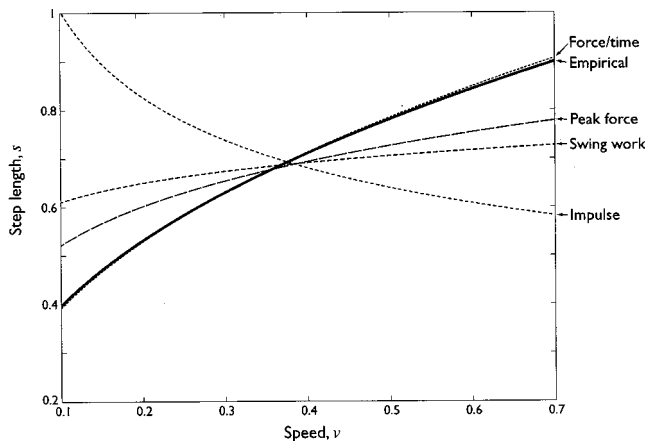


Fig. 4 Predicted speed-step length relationships for the Anthropomorphic Model (AM), compared with the empirical curve $E \sim v^{0.42}$ (thick solid line). Each prediction is based on a metabolic cost per distance E composed of toe-off work plus a hypothesized cost (listed in legend) for tuning the swing leg. The relative weight of these costs, c , was adjusted to make the predictions intersect the empirical curve at about $v=0.35$. The best prediction was made by the cost of force/time (force divided by duration of step). Swing work and peak force tend to modulate step frequency rather than length. The cost of impulse (integral of force over time) favors shorter step lengths at high speeds, opposite to observed behavior.

empirical curve (see Fig. 1) at a speed near $v=0.35$. Then a logarithmic regression was applied to find best-fit speed exponents, and the sensitivity of this exponent to the chosen value of c was also calculated.

Results show that there were substantial qualitative similarities between the results from all three models, although the quantitative differences were sometimes significant (see Table 1). The ISM, when compared with SM, was able to accurately predict the

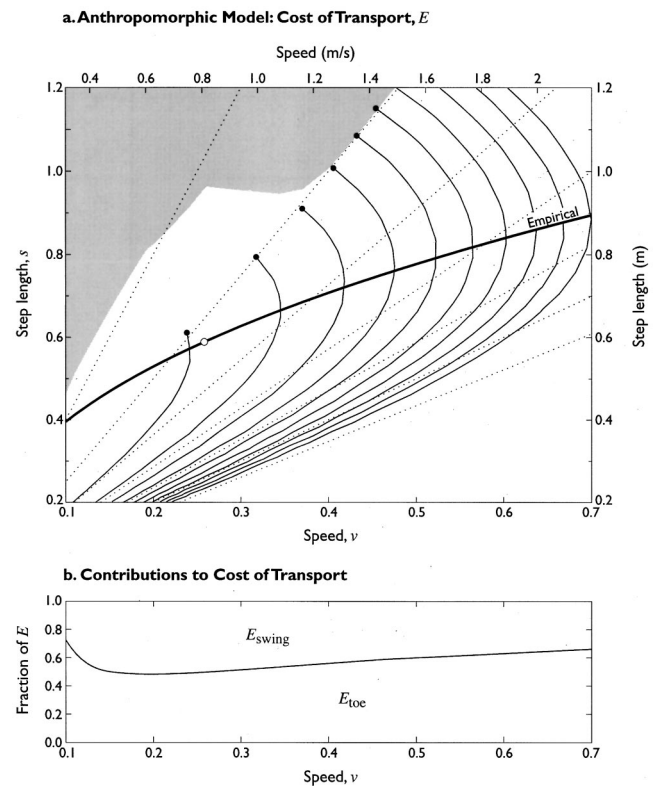


Fig. 5 (a) Predicted contours of metabolic cost of transport, E , versus speed, v , and step length, s , assuming a cost proportional to a weighted sum of mechanical work performed at toe off and the force/time associated with tuning the swing leg of the Anthropomorphic Model (AM). Constant energy contours shown (thin solid lines) are in constant increments. For any given speed, E is minimized at a step length close to $s = v^{0.42}$ (thick solid line). There is also a sharp increase in E with increasing step frequency (dotted lines denote contours of constant step frequency, as in Fig. 3). Units of bottom and left sides are dimensionless; units on top and right sides are for a model with leg length 1 m. (b) Relative contribution to the cost of transport between the toe-off impulse, E_{toe} , and the cost of force/time needed to actuate the swing leg, E_{swing} for preferred gaits. At least 50 percent of the overall cost of transport is due to the propulsive toe-off work.

ordering of speed exponents for various cost models, although with a significant but fairly consistent degree of underprediction. With the addition of more realistic inertial parameters in AM, most of the predicted β were similar to those from SM. Most of the relative sensitivities of predicted β to choice of proportionality c were found to be low, much less than unity (see Table 1). For example, a 10 percent change in c causes only a 1.5 percent change in β for the force/time model, consistent with the qualitative expectation from the ISM.

Results from the AM show that the cost of force/time best predicts preferred step length–speed relationship (see Fig. 4). The peak force, swing work, and impulse costs tend to favor shorter steps than those observed in humans at higher speeds. The corresponding energy contours (Fig. 5(a)) resemble those observed empirically (Fig. 1) near the preferred domain of step lengths, although they diverge at low step frequencies, where the gaits are unstable. For preferred gaits, the overall cost of transport is weakly dominated by E_{toe} , which is responsible for about 60 percent of the energy consumption for most speeds (Fig. 5(b)).

5 Discussion

The modified passive dynamic walking models are useful for ruling out a number of metabolic cost hypotheses. It is clear that the impulse (integral of force over time) cost makes an incorrect prediction of decreasing step lengths with speed. The remaining costs produce relatively good predictions, although swing work and peak force costs both make predictions that are somewhat low.

Perhaps the most surprising finding is that the minimization of swing work is not the most favorable determinant of step length. It places an overly low penalty on increasing step frequency relative to that of increasing step length—cost per distance increases approximately with the second power of frequency—resulting in a low prediction for the speed exponent β . This is true despite the fact that the amount of swing work is quite significant in the Anthropomorphic Model. For the springlike model walking at a speed of $v=0.51$ and a step length of $s=0.77$, swing work accounts for 64 percent of the total mechanical work if no energy shortage is allowed. This figure decreases with shorter burst durations, nearing a still significant 30 percent as the duration approaches zero. Of course, if sufficiently compliant tendons are included, the hip muscles could act isometrically, leaving all of the swing work to be performed conservatively. Nevertheless our model shows that any quantity of swing leg work is insufficient to predict the speed–step length relationship.

What appears to be a more significant determinant of step length is the cost of producing force over short times. This quantity increases sharply, approximately with step frequency raised to the third power, making it more comparable with the mechanical work cost of increasing step length. The distribution between step length and frequency is more equitable with increasing speed, resulting in a predicted speed exponent $\beta=0.43$, which is very close to the observed value.

Minimization of swing work does, however, appear to be a determinant of burstlike hip muscle activity. Bursts with a short duty factor tend to minimize both swing work and impulse magnitude. Force/time is very small except for very short bursts, where the cost increases sharply [22]. Given a short duty factor, the determining cost of increasing step frequency during the leg swing is dominated by force/time rather than swing work or other possible costs.

The force/time model is attractive for several reasons. It predicts a rapidly increasing metabolic cost for high step frequencies, consistent with empirical observations (see Figs. 1 and 5(a)). Physiologically, it is consistent with the nature of hip muscle activity, which appears comparable in duration to the knee muscle activity observed by Kram and Taylor [23] during running. The same model gave quite accurate predictions for metabolic energy consumption in the latter case, and could therefore be applicable

to walking as well, except that the analogy is to forced oscillation with a rotational spring at the hip. It is also encouraging to see that it predicts a speed exponent of $\beta=0.43$, which falls within the empirical range and is better than would be expected for the simplistic models presented here.

It is likely that both mechanical work and force/time costs apply to toe-off and swing leg tuning, but differing contexts cause only one cost to dominate in each case. When applied to tuning of the swing leg, the desire to minimize swing work translates into short bursts of hip muscle activity. Work and force/time are presumably balanced against each other, but when step frequency increases, the slope of force/time increases more sharply and therefore dominates when determining the speed exponent β . In the case of toe-off, work should dominate force/time. In humans, the ankle plantarflexes significantly during toe-off, so that the majority of the energy provided is in terms of work. This is in contrast to the hip muscles, which are activated when the swing leg is at its lowest angular velocity and therefore consume energy without performing much work. Another reason is that the longer tendons of the ankle muscles should tend to increase burst duration and lower force/time. The short tendons at the hips promote maximal muscle-tendon stiffness and therefore small duty factors and short bursts. In any case, the cost of force/time follows nearly the same power law when applied to either toe-off and swing leg tuning, so that the predicted speed exponent should be the same when a single combined cost is used.

One significant unexplained limitation of the present models is the overly low energetic cost predicted for very low step frequencies. There should be a rapidly increasing cost in that regime, because the energy contours in the upper left-hand corner of Fig. 1 are nearly parallel with lines of constant frequency. However, the predicted energy contours do not place a similar cost in that same region (Fig. 5(a)). All of the E_{swing} models considered here have zero energetic cost at zero stiffness k , and we have not as yet proposed a cost for negative stiffness, which would be necessary to provide sufficiently low step frequencies but may likely be achieved by a different mechanism than hip muscle bursts. In any event, it appears to be less costly metabolically to use the hip muscles to speed rather than to retard the swinging of the legs. Inclusion of a proper model of negative stiffness would be expected to improve the shape of the predicted energy contours, but should have relatively little effect on the predicted speed exponent β .

A curious feature of the simulated low frequency gaits is that the system loses passive stability and becomes very unstable (e.g., step-to-step eigenvalues of magnitude greater than 15) at very modest levels of negative stiffness (see Fig. 5(a)). This introduces another potential energetic cost, because the central nervous system would have to provide stabilization in such cases. Though purely speculative at the moment, it is possible that there is a significant metabolic advantage to placing the legs in a passively stable regime, because the need for active control against small disturbances is obviated [24]. The apparent cost of negative stiffness at low setup frequencies might therefore be due to a high cost of active control.

There are several other limitations to the model. Most significant is the lack of a finite double support phase. The duty factor of double support changes with speed [4], which may have an effect on the metabolic cost of the toe-off impulse. Our model also does not include a trunk, which from the studies of McGeer [18] has little effect on the motion of the legs but nevertheless requires stabilization. Other more realistic additions include the knee joints [19] and three-dimensional motion [24], both of which would be expected to add to overall mechanical energy expenditure.

It is important to note that the metabolic cost of locomotion can ultimately only be understood through careful experimental rather than theoretical, study. Physiological understanding of the cost of force/time is incomplete [25]. However, the models presented here may be useful for making conceptual predictions that may

guide future experiments. These include predictions: (1) that the metabolic cost of walking may be separable into components related to mechanical work done by the stance leg at toe-off, and to forced motion of the swing leg; (2) that the metabolic cost of swinging the leg may be dominated by the peak muscle force divided by the burst duration; (3) that these bursts may in turn be determined by the desire to minimize swing work; and (4) that there may be additional metabolic costs for controlling a passively unstable system.

Acknowledgments

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Nomenclature

c	= constant of proportionality of E_{swing} relative to E_{toe} in overall cost
E	= total cost of transport, metabolic energy normalized by body weight and distance traveled
E_{toe}	= metabolic cost of transport associated with work done on center of mass/stance leg at toe-off
E_{swing}	= metabolic cost of transport associated with swing leg motion
g	= gravitational constant
k	= torsional spring constant for swing leg spring
l	= leg length
M	= point mass representing pelvis/torso
m	= point mass representing foot mass
P	= toe-off impulse
s	= step length
t	= dimensionless time
v	= forward walking speed
W_{toe}	= mechanical work per distance performed by toe-off actuation of the stance leg
W_{swing}	= mechanical work per distance performed by a hip torque on the swing leg
α	= initial condition for stance leg angle
β	= speed exponent ($s \sim v^\beta$)
κ	= metabolic cost exponent for ω
λ	= metabolic cost exponent for P
τ	= step period
ω	= natural frequency of swing leg

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